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While Gauss refers to two earlier synthetic constructions of Paucker,[1] as the only previous ones which he knew had been publicly discussed, he remarks that that of Erchinger is "different and carried through more in the spirit of pure geometry."

Stäckel reviews the discussions of the division of a circle into n equal parts, by means of imaginary quantities and the solution of the equation $x^n - 1 = 0$, by Cotes,[2] De Moivre,[3] Euler,[4] and Vandermonde,[5] and the development of their ideas by Gauss in connection with the problem of constructing a regular inscribed polygon.

PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them.* If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

2843. Proposed by E. H. MOORE, University of Chicago.

Show that the maximum of the absolute value of

$$2(a + ib)(x + iy) + i(a + ib)(z + iw) + i(c + id)(x + iy),$$

where $i = \sqrt{-1}$ and a, b, c, d, x, y, z, w are real numbers for which

$$a^2 + b^2 + c^2 + d^2 = x^2 + y^2 + z^2 + w^2 = 1,$$

is $1 + \sqrt{2}$. Study the locus of point-pairs $P = (a, b, c, d)$, $Q = (x, y, z, w)$ of the unit-sphere in real four-space for which this absolute value assumes its maximum value.

2844. Proposed by J. L. RILEY, Stephenville, Texas.

Decompose into simple fractions the number $\frac{6099380351}{1271808720}$ (Gauss, *Disq. Arith.*, *Werke*, vol. 1, pp. 386–387).

2845. Proposed by E. L. POST, Princeton University.

Prove that if y_x is a solution of the functional equation

$$y_x = \frac{y_{x+1}^2}{x} + y_{x+1}$$

for positive integral values of x with $y_x > 0$, then

$$\lim_{x \to \infty} y_x \log x = 1.$$

2846.

Find the entire volume within the surface $x^{1/2} + y^{1/2} + z^{1/2} = a^{1/2}$. (W. A. Granville, *Elements of Differential and Integral Calculus*, revised ed., 1911, p. 420.)

This equation, rationalized, is the equation of Steiner's quartic surface, every tangent plane to which cuts it in two conics. (Cf. Salmon-Rogers, *Analytic Geometry of Three Dimensions*, 5th ed., vol. 2, 1915, pp. 171, 201, 207, 213f. Also C. M. Jessop, *Quartic Surfaces* 1916, Chapter 7.)

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[1] (1) "Geometrische Verzeichnung des regelmässigen 17-Ecks und 257-Ecks in d. Kreis," *Jahresverhandl. d. kurländischen Gesellschaft für Litteratur und Kunst*, Mitau, Band 2, 1822. (2) *Die ebene Geometrie der geraden Linie und des Kreises.* Königsberg, 1823, p. 187. Paucker is also the author of: (3) *De divisione geometrica peripheriae circuli in XVII partes œquales*, Königsberg, 1817.

[2] R. Cotes, *Harmonia mensurarum, sive analysis et synthesis per rationum et angulorum mensuras promota.* Cambridge, 1722.

[3] A. De Moivre, *Miscellanea analytica*, London, 1730.

[4] L. Euler, *Introductio in analysin*, Lausanne, 1748, especially t. 1, cap 8: De quantitatibus transcendentibus ex circulo ortis.

[5] C. A. Vandermonde, "Remarques sur les problèmes de situation," *Histoire de l'Acad.*, année 1771, Paris 1774; *Mémoires*, p. 566. "Sur la résolution des équations," p. 365.

2847. Proposed by B. F. FINKEL, Drury College.

Convert $+\sqrt{R^2 - x^2}$ into a Fourier series.

2848. Proposed by the late L. G. WELD.

A particle is attracted by a finite, uniform, material right line. Define its path, considering: (*a*) that the path is the envelope of the instantaneous lines of resultant attraction, as when the particle moves in a highly viscous medium (*i.e.*, without inertia); (*b*) that the particle moves freely (with inertia).

2849. Proposed by S. A. COREY, Des Moines, Iowa.

In the *Annals of Mathematics*, for April, 1911, Professor Byerly has given a method of approximately representing $f(x)$ in terms of $F_1(x)$, $F_2(x)$, $\cdots$, $F_r(x)$, in an interval $x_0 \leqq x \leqq x_1$. If $F_n(x) = x^n$ and if the mth derivative of $f(x)$ is zero when $x = 0$ for $m > r$, it is evident that Byerly's development becomes exact, and, therefore, identical with Maclaurin's development in the interval $0 \leqq x \leqq x_1$, provided $f(0) = 0$. If $f(0) \neq 0$, it is necessary to replace $f(x)$ by $f(x) - f(0)$ in Byerly's development. The coefficient of x^n in this development is D_n/D, where the D_n and D are certain determinants with elements of the type

$$A_n = \frac{x_1^{2n+1}}{2n+1}, \qquad B_{s,t} = \frac{x_1^{s+t+1}}{s+t+1}, \qquad C_n = \int_0^{x_1} f(x)x^n dx.$$

Prove that as r becomes infinite, D_n/D approaches $f^{[n]}(0)/n!$, the corresponding coefficient in the Maclaurin development, whenever $f(x)$ is analytic, and that any value of x within the range of convergence of the Maclaurin development may be substituted for x_1 without altering the value of D_n/D.

SOLUTIONS OF PROBLEMS.

416 (Algebra) [1914, 156; 1919, 312] Proposed by C. E. FLANAGAN, Wheeling, W. Va.

The sides of a given rectangle are a and b in which a rectangle is to be inscribed one of whose sides is c. Find the other side, using Euler's rule for quartics.

I. SOLUTION AND DISCUSSION BY OTTO DUNKEL, Washington University.

Let the given rectangle be $RSS'R'$ such that $RS = R'S' = a$ and $RR' = SS' = b$, and let $PP'Q'Q$ be a rectangle inscribed in it such that $QP = Q'P' = c$ and $PP' = QQ' = x$. Suppose further that the vertex P lies on RS, the vertex P' on SS' and so on in order. Let $SP = m$ and $SP' = n$; then from the four similar triangles in the figure, pairs of which are equal, we have at once

$$(1) \qquad \frac{n}{x} = \frac{a-m}{c}, \qquad \frac{m}{x} = \frac{b-n}{c}, \qquad n^2 + m^2 = x^2,$$

and from the first two equations follow

$$(2) \qquad \begin{gathered} n + m = \frac{(a+b)x}{c+x}, \qquad n - m = \frac{(a-b)x}{c-x}, \\ n(c^2 - x^2) = (ac - bx)x, \qquad m(c^2 - x^2) = (bc - ax)x. \end{gathered}$$

Squaring and adding the equations in the first line of (2) and using the third equation in (1) we obtain

$$(3) \qquad 2 - \left(\frac{a+b}{c+x}\right)^2 - \left(\frac{a-b}{c-x}\right)^2 = 0.$$

It will be seen that the form of the equation above is convenient for separating the roots. On clearing of fractions it becomes

$$(3') \qquad x^4 - (a^2 + b^2 + 2c^2)x^2 + 4abcx - c^2(a^2 + b^2 - c^2) = 0.$$

The other side x of the inscribed rectangle must satisfy this equation, but in order to say that a given root of this equation determines the remaining side of an *inscribed* rectangle it must be shown that this root gives suitable values of m and n for such a rectangle.